Vivekananda
College of Engineering & Technology

Question and Answers on

10CS762 /10IS762

Digital Image Processing

UNIT- 8 : MATHEMATICAL MORPHOLOGY

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Question and Answers

Mathematical Morphology

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Q1. Define morphology and its basics.

Definition

Mathematical Morphology is a tool for extracting image components that are useful for representation and description. It is a set-theoretic method of image analysis providing a quantitative description of geometrical structures.

General Properties

Morphological operations are based on simple expanding and shrinking operations. It simplifies images, and quantifies and preserves the main shape characteristics of objects. The primary application of morphology occurs in binary images, though it is also used on grey level images.

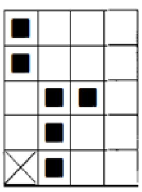
How it works?

A morphological transformation Ψ is given by the relation of the image (point set X) with another small point set B called a structuring element. B is expressed with respect to a local origin O (called the representative point).

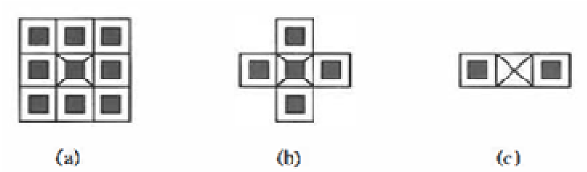
To apply the morphological transformation Ψ(X) to the image X, move structuring element B systematically across the entire image.

Assume that B is positioned at some point in the image; the pixel in the image corresponding to the representative point ***O*** of the structuring clement is called the current pixel.

The result of the relation (which call be either zero or one) between the image X and the structuring element B in the current position is stored in the output image in the current image pixel position.



A point set example.



Typical structuring elements.

Q2. List the applications of morphology

Morphological operations are used predominantly for the following purposes:

1. Image pre-processing (noise filtering, shape simplification).
2. Enhancing object structure (skeletonising, thinning, thickening, convex hull, object marking) .
3. Segmenting objects from the background.
4. Quantitative description of objects (area, perimeter, projections, Euler-Poincare characteristic).

Q3. Explain in detail, the four basic principles of morphology

A morphological transformation Ψ is called quantitative if and only if it satisfies four basic principles: Compatibility with translation, Compatibility with change of scale, Local knowledge and Upper semi-continuity

1. Compatibility with translation

Let the transformation Ψ depend on the position of the origin O of the co-ordinate system. Transformation is denoted by Ψo . If all points are translated by the vector -h, it is expressed Ψ-h. The compatibility with translation principle is given by

$$\Psi_{\mathcal{O}}(X_h) = \big(\Psi_{-h}(X)\big)_h \,.$$

If Ψ does not depend on the position of the origin O , then the compatibility with translation: principle reduces to invariance under translation

$$\Psi(X_h) = \big(\Psi(X)\big)_h$$

2. Compatibility with change of scale

Let λX represent the homothetic scaling of a point set X (i.e., the co-ordinates of each point of the set are multiplied by some positive constant λ). This is equivalent to change of scale with respect to origin.

Let Ψλ denote a transformation that depends on the positive parameter λ (change of scale). Compatibility with change of scale is given by

$$\Psi_\lambda(X) = \lambda\,\Psi\left(\frac{1}{\lambda}\,X\right)$$

If Ψ does not depend on the scale λ, then compatibility with change of scale reduces to invariance to change of scale

$$\Psi(\lambda\,X) = \lambda\Psi(X)$$

3. Local knowledge

The morphological transformation Ψ satisfies the local know ledge principle if for any bounded point set Z' in the transformation Ψ (X) there exists a bounded set Z, knowledge of which is sufficient to provide Ψ.

The local knowledge principle may be written symbolically as

$$\big(\Psi(X \cap Z)\big) \cap Z' = \Psi(X) \cap Z'$$

4. Upper semi-continuity

The upper semi-continuity principle says that the morphological transformation does not exhibit any abrupt changes.

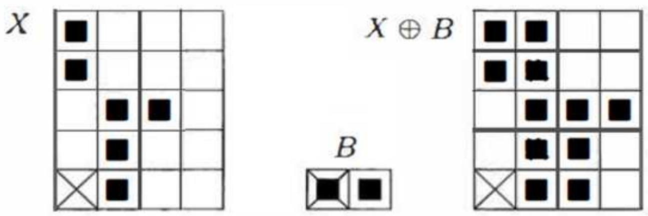
Q4. Explain binary dilation and erosion.

Dilation: The dilation $X \oplus B$ is the point set of all possible vector additions of pairs of elements, one from each of the sets A and B

$$X \oplus B = \{p \in \mathcal{E}^2 : p = x + b,\ x \in X \text{ and } b \in B\}.$$

Example:

$$X = \{(1,0),(1,1),(1,2),(2,2),(0,3),(0,4)\},$$
$$B = \{(0,0),(1,0)\},$$
$$X \oplus B = \{(1,0),(1,1),(1,2),(2,2),(0,3),(0,4),(2,0),(2,1),(2,2),(3,2),(1,3),(1,4)\}$$



Note: In the above example in a point *p(x,y)*, x denotes column number starting with 0 and y denotes row number starting with 0

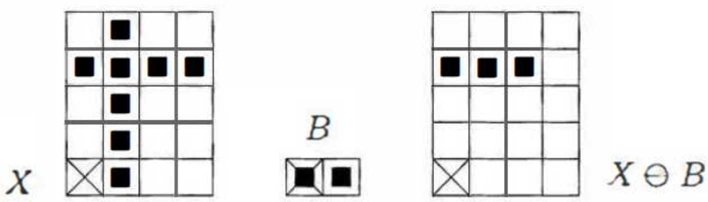
Erosion: Erosion combines two sets using vector subtraction of set elements and is the dual operator of dilation.

$$X \ominus B = \{p \in \mathcal{E}^2 : p = x + b \in X \text{ for every } b \in B\}$$

This formula says that every point p from the image is tested. The result of the erosion is given by those points p for which all possible p + b are in X.

Example:

$$X = \{(1,0),(1,1),(1,2),(0,3),(1,3),(2,3),(3,3),(1,4)\}$$
$$B = \{(0,0),(1,0)\},$$
$$X \ominus B = \{(0,3),(1,3),(2,3)\}.$$



Note: Neither erosion nor dilation is an invertible transformation

Q5. Explain Hit and Miss Transformation

The hit-or-miss transformation is the morphological operator for finding local patterns of pixels, where local means the size of the structuring element. It is a variant of template matching that finds collections of pixels with certain shape properties (such as corners or border points).

The transform involves looking for pixel positions where one component lies within the set of black pixels, and the other lies completely without.

An operation may be denoted by a pair of disjoint sets B=(B1,B2), called a composite structuring element.

The **hit-or-miss** transformation $\otimes$ is defined as

$$X \otimes B = \{x : B_1 \subset X \text{ and } B_2 \subset X^c\}$$

where B_1 – set of foreground pixels(1's), B_2 – set of background pixels (0's)

It may be expressed using erosions and dilations as well

$$\begin{aligned} X \otimes B &= (X \ominus B_1) \cap (X^c \ominus B_2) \\ &= (X \ominus B_1) \setminus (X \oplus \breve{B}_2) \end{aligned}$$

Set **difference** is defined by $X \setminus Y = X \cap Y^c$

Application: Corner Detection

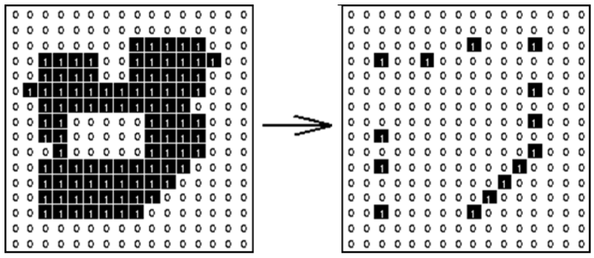
Special types of structuring elements are used to detect four types of corners. The original image and the result is shown below.

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	



Q6. Discuss the concept of opening and closing

Erosion followed by dilation is called ***opening***. Opening removes thin connections, small protrusion in the image.

$$X \circ B = (X \ominus B) \oplus B .$$

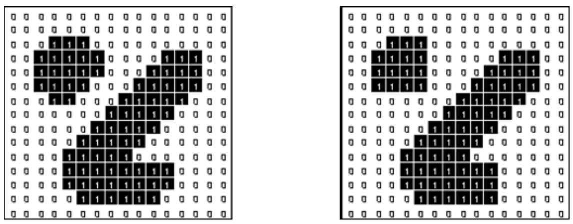
Dilation followed by erosion is called ***closing.*** Closing removes holes in the image.

$$X \bullet B = (X \oplus B) \ominus B .$$

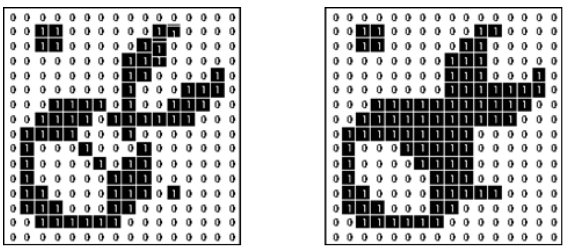
If an image X is unchanged by opening with the structuring element B, it is called open with respect to B. Similarly, if an image X is unchanged by closing with B, it is called closed with respect to B.

Illustration:

Result of opening by 3x3 square structuring element



Result of closing by 3x3 square Structuring element



Properties

Opening is anti-extensive ($X \circ B \subseteq X$)

closing is extensive ($X \subseteq X \bullet B$).

Opening and closing, like dilation and erosion, are dual transformations

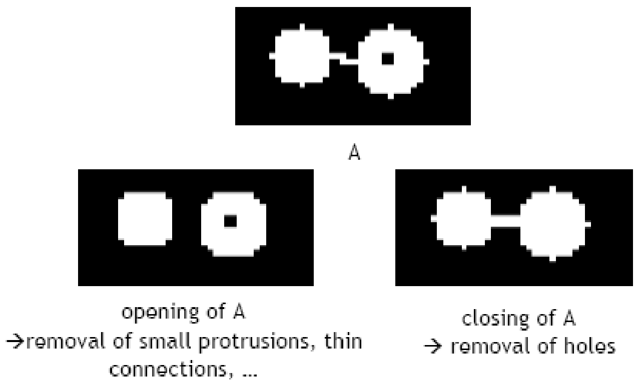
$$(X \bullet B)^C = X^C \circ \breve{B} .$$

Another significant fact is that iteratively used openings and closings are idempotent, Reapplication of these transformations does not change the previous result.

$$X \circ B = (X \circ B) \circ B ,$$
$$X \bullet B = (X \bullet B) \bullet B .$$

Application

1. Preprocessing fingerprint images.
2. Closing connects objects that are close to each other, fills up small holes, and smoothes the object outline by filling up narrow gulfs
3. Opening and closing with an isotropic structuring element is used to eliminate specific image details smaller than the structuring element the global shape of the objects is not distorted.



Q7. Describe the role of structuring element in mathematical morphology.

The result of mathematical morphology is determined by size/shape of the structuring element.

In dilation the image is expanded as per the size and shape of the structuring element. As size increases, dilation results in filling bigger gaps and holes.

In erosion the image is shrinked relative to the size and shape of the structuring element. As size increases, erosion results in removing larger protrusion.

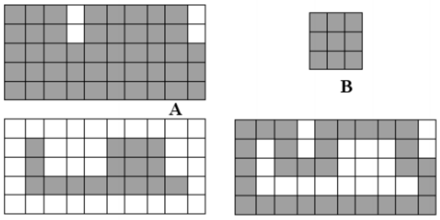
When used by a hit-or-miss transform, usually the structuring element is a composite of two disjoint sets (two simple structuring elements), one associated to the foreground, and one associated to the background of the image to be probed.

Q8. Explain boundary extraction using morphological operators.

The boundary of a set A, denoted by β(A), can be obtained by first eroding A by B and then performing the set differences between A and its erosion. That is,

$\beta(A) = A - (A \Theta B)$, where B is a suitable structuring element(usually 3x3 matrix),

Illustration



AΘB β(A)

Q9. What do you mean by skeletanization? Explain thinning by structural elements.

Skeletonization is a process for reducing foreground regions in a binary image to a skeletal residue that largely preserves the extent and connectivity of the original region while throwing away most of the original foreground pixels.

The skeleton can be produced by successive morphological **thinning**.

Thinning is a morphological operation that is used to remove selected foreground pixels from binary images. The thinning operation is related to the hit-and-miss transform. The thinning of an image I by a structuring element J is:

$$thin(I,J) = I - hit\text{-}and\text{-}miss(I,J)$$

where the subtraction is a *logical subtraction* defined by $X - Y = X \cap \text{NOT } Y$.

Repeated morphological thinning operation continuously erodes away pixels from the boundary (while preserving the end points of line segments) until no more thinning is possible, at which point what is left approximates the skeleton.

Illustration

Consider the Structuring elements.

The origin of the structuring element is at the center.

0	0	0
	1	
1	1	1

	0	0
1	1	0
	1	

At each iteration, the image is thinned by the left structuring element, and then by the right one, and then with the remaining six *90°* rotations of the two elements. The process is repeated in cyclic fashion until none of the thinnings produces any further change. As usual,

Binary Image

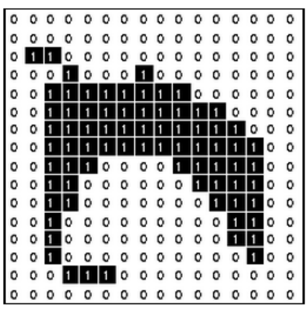
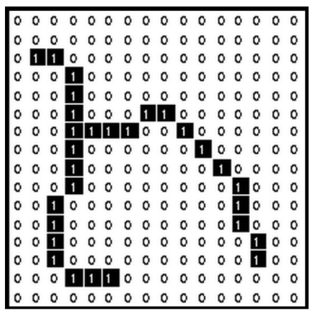


Image after skeletanization



Q10. How Region/Hole filling is achieved through morphology?

A hole may be defined as the background region surrounded by a connected border of foreground pixels. Hole filling is achieved by dilation, complementation and intersection.

Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole). Given a point in each hole, the objective is to fill all the holes with 1s.

We begin by forming an array, X_0, of 0s (the same size as the array containing A), except at the locations in X_0 corresponding to the given point in each hole, which we set to 1. Then, the following procedure fills all the holes with 1s:

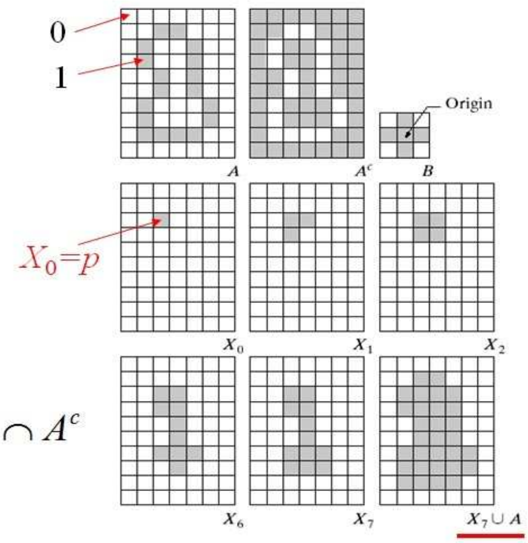
$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1, 2, 3, \ldots \tag{9.5-2}$$

where B is the symmetric structuring element in Fig. 9.15(c). The algorithm terminates at iteration step k if $X_k = X_{k-1}$. The set X_k then contains all the filled holes. The set union of X_k and A contains all the filled holes and their boundaries.

Illustration

a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



$$X_k = (X_{k-1} \oplus B) \cap A^c$$

Q11. Describe Umbra, Top surface

Top Surface

Let $A \subseteq \mathcal{E}^n$ and the support $F = \{x \in \mathcal{E}^{n-1}$ for some $y \in \mathcal{E}$, $(x, y) \in A\}$. The **top surface** of A, denoted by $T[A]$, is a mapping $F \to \mathcal{E}$ defined as

$$T[A](x) = \max\{y,\ (x, y) \in A\}\,.$$

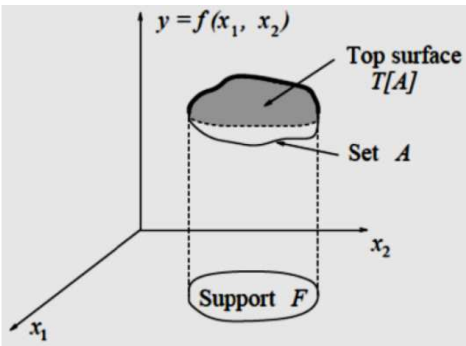
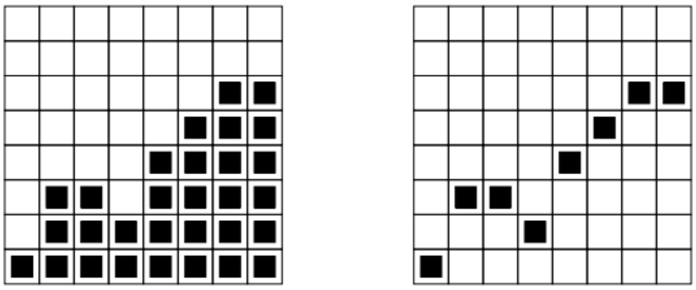


Figure 13.12: Top surface of the set A corresponds to maximal values of the function $f(x_1, x_2)$.

Example

$$T(R) = \{(x, y) : y \geq z \text{ for all } (x, z) \in R\}$$



R $T(R)$

Umbra

Formally, let $F \subseteq \mathcal{E}^{n-1}$ and $f : F \to \mathcal{E}$. The **umbra** of f, denoted by $U[f]$, $U[f] \subseteq F \times \mathcal{E}$, is defined by

$$U[f] = \{(x, y) \in F \times \mathcal{E},\ y \leq f(x)\}. \qquad (13.37)$$

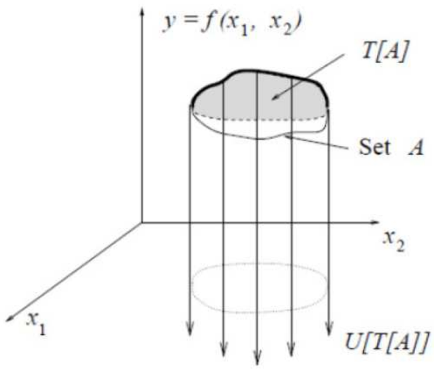
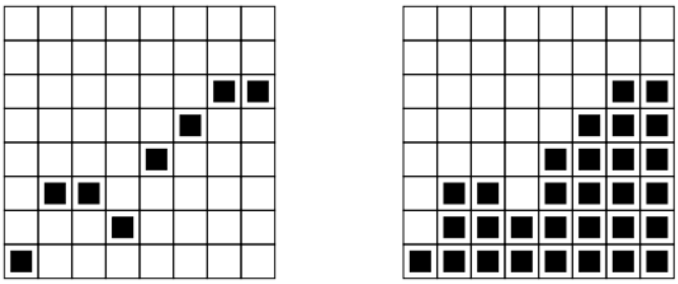


Figure 13.13: Umbra of the top surface of a set is the whole subspace below it.

Example

$$U(f) = \{(x, y) : y \leq f(x)\}$$



f $U(f)$

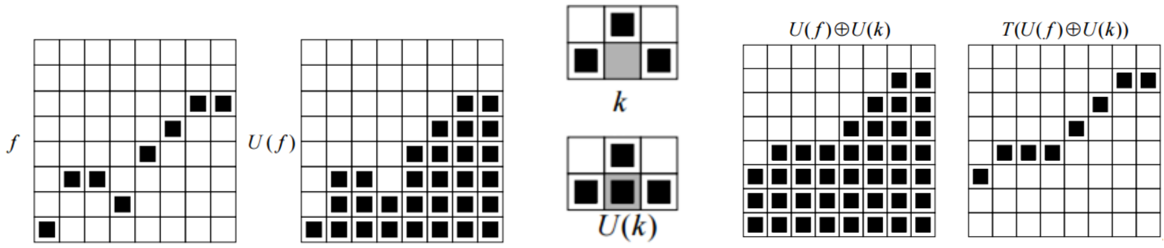
Q12. Explain how erosion and dilation can be applied to a grayscale image?

GRAY SCALE DILATION

We can now define the gray-scale dilation of two fınctions as the top su face of the dilation of their umbras. Let $F, K \subseteq \mathcal{E}^{n-1}$ and $f : F \to \mathcal{E}$ and $k : K \to \mathcal{E}$. The dilation $\oplus$ of f by k, $f \oplus k : F \oplus K \to \mathcal{E}$ is defined by

$$f \oplus k = T\{U[f] \oplus U[k]\} . \tag{13.38}$$

<u>Illustration</u> (f is 1-dimentional function)



GRAY SCALE EROSION

The definition of **gray-scale erosion** is analogous to gray-scale dilation. The gray-scale erosion of two functions (point sets)

1. Takes their umbras.
2. Erodes them using binary erosion.
3. Gives the result as the top surface.

Let $F, K \subseteq \mathcal{E}^{n-1}$ and $f : F \to \mathcal{E}$ and $k : K \to \mathcal{E}$. The **erosion** $\ominus$ of f by k, $f \ominus k : F \ominus K \to \mathcal{E}$ is defined by

$$f \ominus k = T\{U[f] \ominus U[k]\} . \tag{13.40}$$

<u>Illustration</u> (f is 1-dimentional function)

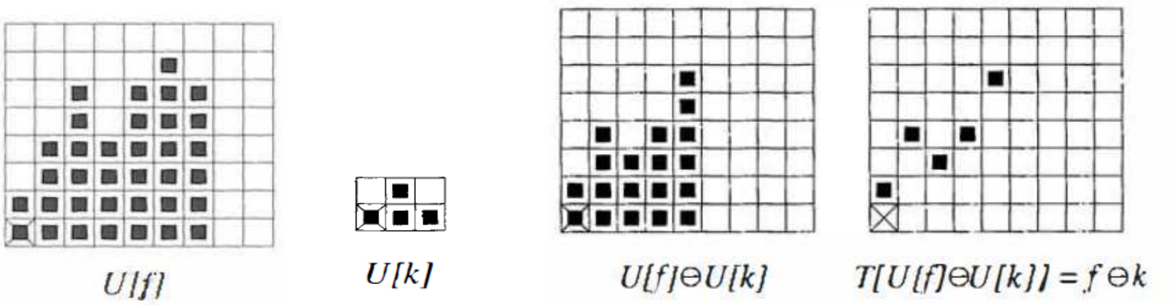


Figure 13.17: 1D example of gray-scale erosion. The umbras of 1D function f and structuring element k are eroded first, $U[f] \ominus U[k]$. The top surface of this eroded set gives the result, $f \ominus k = T[U[f] \ominus U[k]]$.

Illustration of Dilation and Erosion on Gray scale images

Dilation is the maximum of all pixels of f that are 'hit' by S. Erosion is the minimum of all pixels of f that are 'hit' by S.

1	1	0	0	1	2
2	2	1	0	0	1
3	3	3	1	0	1
3	3	1	0	1	2
3	1	0	1	2	3
1	0	1	2	3	4

Image

	0	
0	0	0
	0	

Structuring element

1	0	0	0	0	1
1	1	0	0	0	0
2	2	1	0	0	0
3	1	0	0	0	1
1	0	0	0	1	2
0	0	0	1	2	3

Result of Erosion

2	2	1	1	2	2
3	3	3	1	1	2
3	3	3	3	1	2
3	3	3	1	2	3
3	3	1	2	3	4
3	1	2	3	4	4

Result of Dilation

Q13. Describe top hat transformation with a diagram.

Top hat transformation is the difference of the original image and the opened image.

It is used as a simple tool for segmenting objects in gray-scale images that differ in brightness from background, even when the background is of uneven gray-scale.

Assume a gray-level image X and a structuring element K. The residue of opening as compared to original image ***X \ (X* o *K)*** constitutes a new useful operation called a top hat transformation. Set **difference** is defined by $X \setminus Y = X \cap Y^c$

The top hat transformation is a good tool for extracting light objects (or dark ones) on a dark (or light) but slowly changing background. Those parts of the image that cannot fit into structuring element K are removed by **opening**. **Subtracting** the opened image from the original provides an image where removed objects stand out clearly. The actual segmentation can be performed by simple **thresholding**.

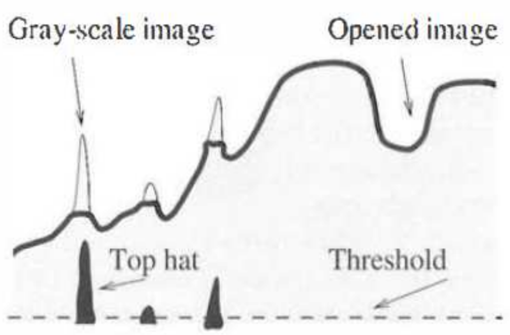


Figure 13.19: The top hat transform permits the extraction of light objects from an uneven background.

Q14. Explain in morphological segmentation and watershed.

General Procedure

Mathematical morphology helps mainly to segment images of texture or images of particles. Morphological particle segmentation is performed in two basic steps: Location of particle markers and watersheds for particle reconstruction.

Marker extraction resembles human behavior when one is asked to indicate objects; the person just points to objects and does not outline boundaries.

The marker of an object or set X is a set M that is included in X. Marker M can computed manually or semi-automatic marking methods. When the objects are marked, they can be grown from the markers, using the watershed transformation which is motivated by the topographic view of images.

Binary Morphological Segmentation using watersheds

Morphological segmentation in binary images aims to find regions corresponding to individual overlapping objects (typically particles).

Each particle is marked first. Ultimate erosion may be used for this purpose or markers may be placed manually.

The next task is to grow objects from the markers; provided they are kept within the limits of the original set and parts of objects are not joined when they come close to each other. The oldest technique for this purpose is called conditional dilation.

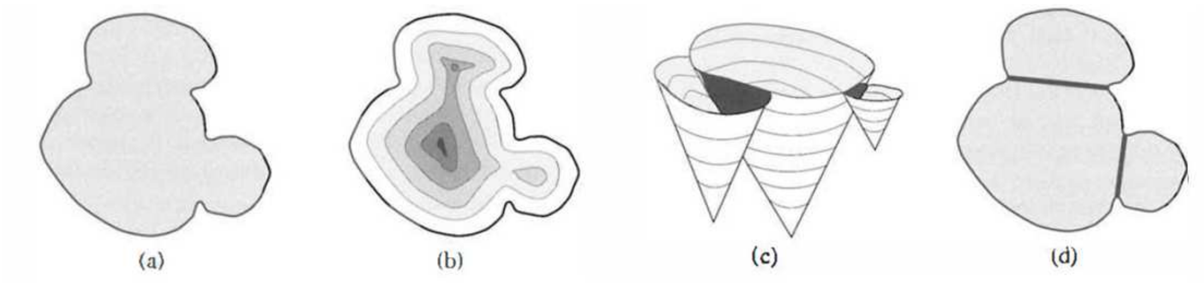


Figure 13.46: Segmentation of binary particles. (a) Input binary image. (b) Gray-scale image created from (a) using the −dist function. (c) Topographic notion of the catchment basin. (d) Correctly segmented particles using watersheds of image (b).

Gray-scale segmentation using watersheds

The markers and watersheds method can also be applied to gray-scale segmentation.

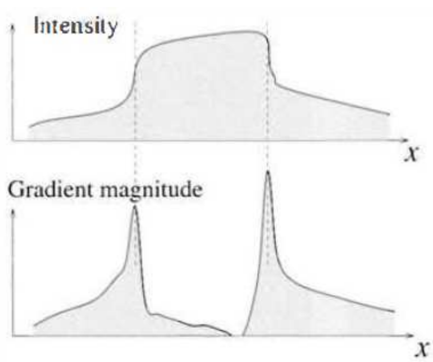
The contour of a region in a gray-level image corresponds to points in the image where gray-levels change most quickly i.e. gradient magnitude is high.

Gradient can be approximated as "Beucher's gradient"

$$\mathrm{grad}(X) = (X \oplus B) - (X \ominus B)$$

The main problem with segmentation via gradient images without markers is over segmentation, i.e the image is partitioned into too many regions.

The watershed segmentation methods with markers do not suffer from over segmentation.



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