Module-2
Regular Grammars

Regular Grammars

A regular grammar *G* is a quadruple (V, Σ, R, S), where:

- *V* is the rule alphabet, which contains nonterminals and terminals,
- Σ (the set of terminals) is a subset of *V*,
- *R* (the set of rules) is a finite set of rules of the form:

  $X \rightarrow Y$,
- *S* (the start symbol) is a nonterminal.

Regular Grammars

In a regular grammar, all rules in R must:

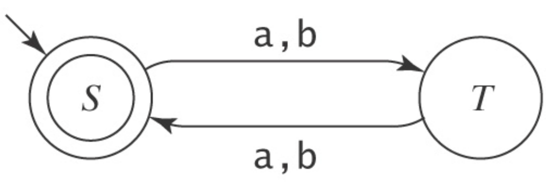
- have a left hand side that is a single nonterminal
- have a right hand side that is:
  - ε, or
  - a single terminal, or
  - a single terminal followed by a single nonterminal.

Legal: $S \rightarrow$ a, $S \rightarrow \varepsilon$, and $T \rightarrow$ aS
Not legal: $S \rightarrow$ aSa and aSa $\rightarrow T$

Regular Grammar Example

$L = \{w \in \{\text{a}, \text{b}\}^* : |w| \text{ is even}\}$

$((\text{aa}) \cup (\text{ab}) \cup (\text{ba}) \cup (\text{bb}))^*$



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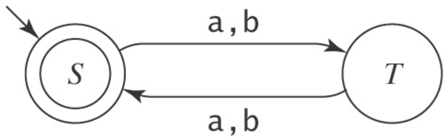

$S \rightarrow \varepsilon$
$S \rightarrow \texttt{a}T$
$S \rightarrow \texttt{b}T$
$T \rightarrow \texttt{a}$
$T \rightarrow \texttt{b}$
$T \rightarrow \texttt{a}S$
$T \rightarrow \texttt{b}S$

Regular Languages and Regular Grammars

Theorem: The class of languages that can be defined with regular grammars is exactly the regular languages.

Proof: By two constructions.

Regular Languages and Regular Grammars

Regular grammar* $\rightarrow$ *FSM:

grammartofsm(G = (V, Σ, R, S)) =

1. Create in M a separate state for each nonterminal in V.
2. Start state is the state corresponding to S .
3. If there are any rules in R of the form $X \rightarrow w$, for some $w \in \Sigma$, create a new state labeled # (Final).
4. For each rule of the form $X \rightarrow w\ Y$, add a transition from X to Y labeled w.
5. For each rule of the form $X \rightarrow w$, add a transition from X to # labeled w.
6. For each rule of the form $X \rightarrow \varepsilon$, mark state X as accepting.
7. Mark state # as accepting.

FSM* $\rightarrow$ *Regular grammar: Similarly.

Example 1 - Even Length Strings

$S \rightarrow \varepsilon$

$S \rightarrow \mathtt{a}T$

$S \rightarrow \mathtt{b}T$

$T \rightarrow \mathtt{a}$

$T \rightarrow \mathtt{b}$

$T \rightarrow \mathtt{a}S$

$T \rightarrow \mathtt{b}S$

Strings that End with aaaa

$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* : w$ ends with the pattern $\mathtt{aaaa}\}$.

$S \rightarrow \mathtt{a}S$
$S \rightarrow \mathtt{b}S$
$S \rightarrow \mathtt{a}B$
$B \rightarrow \mathtt{a}C$
$C \rightarrow \mathtt{a}D$
$D \rightarrow \mathtt{a}$

Strings that End with aaaa

$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* : w$ ends with the pattern $\mathtt{aaaa}\}$.

$S \rightarrow \mathtt{a}S$
$S \rightarrow \mathtt{b}S$
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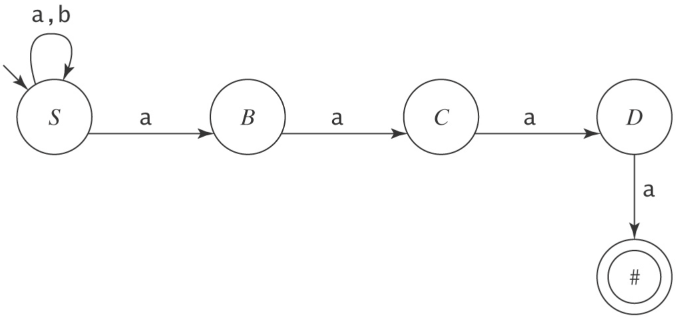
$D \rightarrow \mathtt{a}$



Example 2 – One Character Missing

$S \rightarrow \varepsilon$
$S \rightarrow \mathtt{a}B$
$S \rightarrow \mathtt{a}C$
$S \rightarrow \mathtt{b}A$
$S \rightarrow \mathtt{b}C$
$S \rightarrow \mathtt{c}A$
$S \rightarrow \mathtt{c}B$

$A \rightarrow \mathtt{b}A$
$A \rightarrow \mathtt{c}A$
$A \rightarrow \varepsilon$
$B \rightarrow \mathtt{a}B$
$B \rightarrow \mathtt{c}B$
$B \rightarrow \varepsilon$

$C \rightarrow \mathtt{a}C$
$C \rightarrow \mathtt{b}C$
$C \rightarrow \varepsilon$